BKNG 1 to 16 October 2024

trades on 16 October 2024	11,358
average trading rate	0.6 per second
presample for the diurnal profile	17 to 30 September 2024
outliers on 16 October 2024, o	985 (8.67%)
price half-life: run / minimizer[1]	3.0 / 4.0 trades
cost of running 3 trades	+0.81%
scale half-life: run / minimizer[2]	240 / 360 s
cost of running 240 s	+0.11%
direction half-life[3]	285 s [212, 300]
Mandelbrot a minimizing QLIKE	14 trades
a within 0.0005 of the minimum	10 to 20
linear $\hat{\sigma}$, before $\rightarrow$ at the largest residual[4]	42% $\rightarrow$ 55%
Huber $\hat{\sigma}$, before $\rightarrow$ peak	33% $\rightarrow$ 45%
median spot volatility, Huber / linear input[5]	16.7 / 18.1 %
hyperbolic against HExp, correlation of logs	0.9986
HExp QLIKE advantage, leave-one-day-out[6]	+0.16295
15-minute signature level	15 to 36 %
Huber signature, 1 min over 1 s, 12 day average	0.98

BKNG over the 12 trading days 1 to 16 October 2024, and on 16 October 2024. [1]the median over k from 2 to 100 of the half-life minimizing the k-trade-ahead Huber forecast loss, and the largest cost over those k of running the common 3 trades. [2]the 1-minute-ahead check loss of the noise scale. [3]the half-life maximizing the mean predictive log-likelihood per scored trade on 16 October 2024, with the range within 10^{-4} of the maximum. [4]the largest Huber residual after 09:45, at 09:56; before is the median over $t^* - 15$ to $t^* - 3$ minutes and at is the maximum over $t^* - 6$ to $t^* + 12$ minutes. [5]over the 12 days, sampled every 300 trades. [6]hyperbolic minus HExp, so positive favours HExp.

	Mean of $\lvert\hat{X}_t - \hat{X}_{t-k}\rvert/\sqrt{k}$ in cents				Sqrt of Mean of $(\hat{X}_t - \hat{X}_{t-k})^2/k$ in cents			
k	median	Huber	linear	trades	median	Huber	linear	trades
1	48.700	39.499	37.719	190.043	138.955	93.362	71.463	424.512
2	43.212	40.555	41.016	154.929	116.819	87.771	73.421	317.282
5	45.861	41.729	42.543	108.620	95.655	77.235	69.716	201.997
10	44.042	40.131	40.650	83.836	80.254	69.962	65.041	151.682
25	37.994	35.765	35.918	60.166	58.781	53.812	53.036	98.313
	ACF of $\hat{X}_t - \hat{X}_{t-1}$ at lag k				Hill tail index of $\lvert\hat{X}_t - \hat{X}_{t-k}\rvert$			
1	−0.29	−0.12	0.06	−0.44	1.83	1.50	4.10	2.42
2	−0.02	−0.01	−0.02	−0.01	2.02	1.69	2.57	2.83
5	−0.02	0.04	0.04	0.12	2.53	2.16	3.29	2.94
10	−0.04	−0.15	−0.02	0.00	2.69	2.63	3.71	3.22
25	0.00	0.00	0.01	0.01	2.83	3.25	5.82	11.84

Table 1: Trade-lag differences, `BKNG`, 16 October 2024.

	Mean of $\frac{\lvert\hat{X}_t - \hat{X}_{t,s}\rvert}{\sqrt{\tau_t - \tau_{t,s}}}$ in cents				Annualized volatility: $\hat{\sigma}_{\text{Annual};s}$			
s	median	Huber	linear	trades	median	Huber	linear	trades
0.1 s	119.38	105.69	103.90	297.93	37.5	24.9	23.6	139.6
1 s	78.90	74.69	75.81	199.12	32.0	21.5	21.9	120.1
5 s	55.29	53.13	55.19	129.29	25.3	19.2	20.9	91.7
10 s	49.37	48.80	51.09	109.35	24.8	20.1	21.8	73.8
1 min	35.18	37.13	38.84	71.23	19.9	18.9	20.2	41.4
15 min	30.82	34.53	36.03	54.47	20.3	20.5	21.2	28.1
	$\text{Cor}(\hat{X}_t - \hat{X}_{t,s}, \hat{X}_{t,s} - \hat{X}_{t,2s})$				Hill tail index of $\lvert\hat{X}_t - \hat{X}_{t,s}\rvert$			
0.1 s	−0.12	−0.06	−0.05	−0.15	2.97	2.56	4.36	6.64
1 s	−0.14	−0.07	0.01	−0.15	2.70	2.57	4.00	54.44
5 s	−0.09	−0.01	−0.03	−0.38	3.45	3.20	3.57	4.10
10 s	−0.27	−0.15	−0.17	−0.44	3.05	2.82	4.12	4.66
1 min	−0.13	−0.08	−0.10	−0.37				

Table 2: Clock-lag differences, `BKNG`, 16 October 2024.

	median	Huber	linear
Mean of $\lvert\hat{\epsilon}_t\rvert$ in cents	119.93	138.75	144.57
s.d. of $\hat{\epsilon}_t$ in cents	292.27	274.05	271.79
Tail index of $\psi(\hat{\epsilon}_t)$	∞	∞	3.99
ACF of $\psi(\hat{\epsilon}_t)$, lag 1	0.19	0.13	0.06
ACF of $\psi(\hat{\epsilon}_t)$, lag 2	0.06	0.01	−0.02
ACF of $\psi(\hat{\epsilon}_t)$, lag 5	0.03	0.00	0.04
ACF of $\psi(\hat{\epsilon}_t)$, lag 10	−0.01	−0.04	−0.02
ACF of $\psi(\hat{\epsilon}_t)$, lag 25	0.00	−0.02	0.01
80% C.I. for $\theta_t - X_t$	—	−69.81,73.49	−76.90,77.78
95% C.I. for $\theta_t - X_t$	—	−134.97,153.70	−152.42,157.13
99% C.I. for $\theta_t - X_t$	—	−269.20,308.30	−323.47,358.56
s.e. of $\theta_t - X_t$ in cents	—	70.21	86.75

Table 3: Residual summaries, `BKNG`, 16 October 2024. Median C.I. and s.e. cells are omitted (price discreteness); the linear s.e. is printed because the Hill index of its score, 3.99, is above two, while the Hill index of the Huber residual itself on this day is 1.90, below two: the two estimates of the noise's index straddle two.

	16 October 2024		12 days, min to max	
	upper	lower	upper	lower
All trades (as in the paper)	1.90	2.19	1.90 to 4.00	1.81 to 4.63
Residual divided by δ_t	5.47	4.59	3.91 to 5.70	2.90 to 6.15
Repricing windows removed	2.63	2.19	2.63 to 4.64	1.89 to 4.63
Calm windows only	2.54	2.25		
Every 10th trade	—	—	— to —	— to —
Six-trade half-life	1.94	2.27	1.94 to 4.45	1.92 to 4.83

Table 4: Hill estimator of the tail index of the Huber filter residual, `BKNG`, 12 analysis days. The index is the mean of Hill estimates over the largest 0.1% to 1% of residuals, each tail separately, which for this stock is an average over 8 to 12 depths a day in row 1; the masked and thinned rows use fewer. Second row: divides the residual by its own threshold δ_t. Third row: drops trades where the filtered price moves more than 6.8 times the day's median δ_t within ±1 second, which is 1651.1 to 2478.6 cents over the days and removes 0% to 1% of a day; the fourth keeps only trades where it moves less than 3.4 times that median within ±5 seconds, 972.8 cents on 16 October 2024, 98% of that day. Fifth row: keeps every 10th trade. Sixth row: doubles the filter's half-life with the thresholds held fixed. The ranges run over the days on which the row can be estimated.

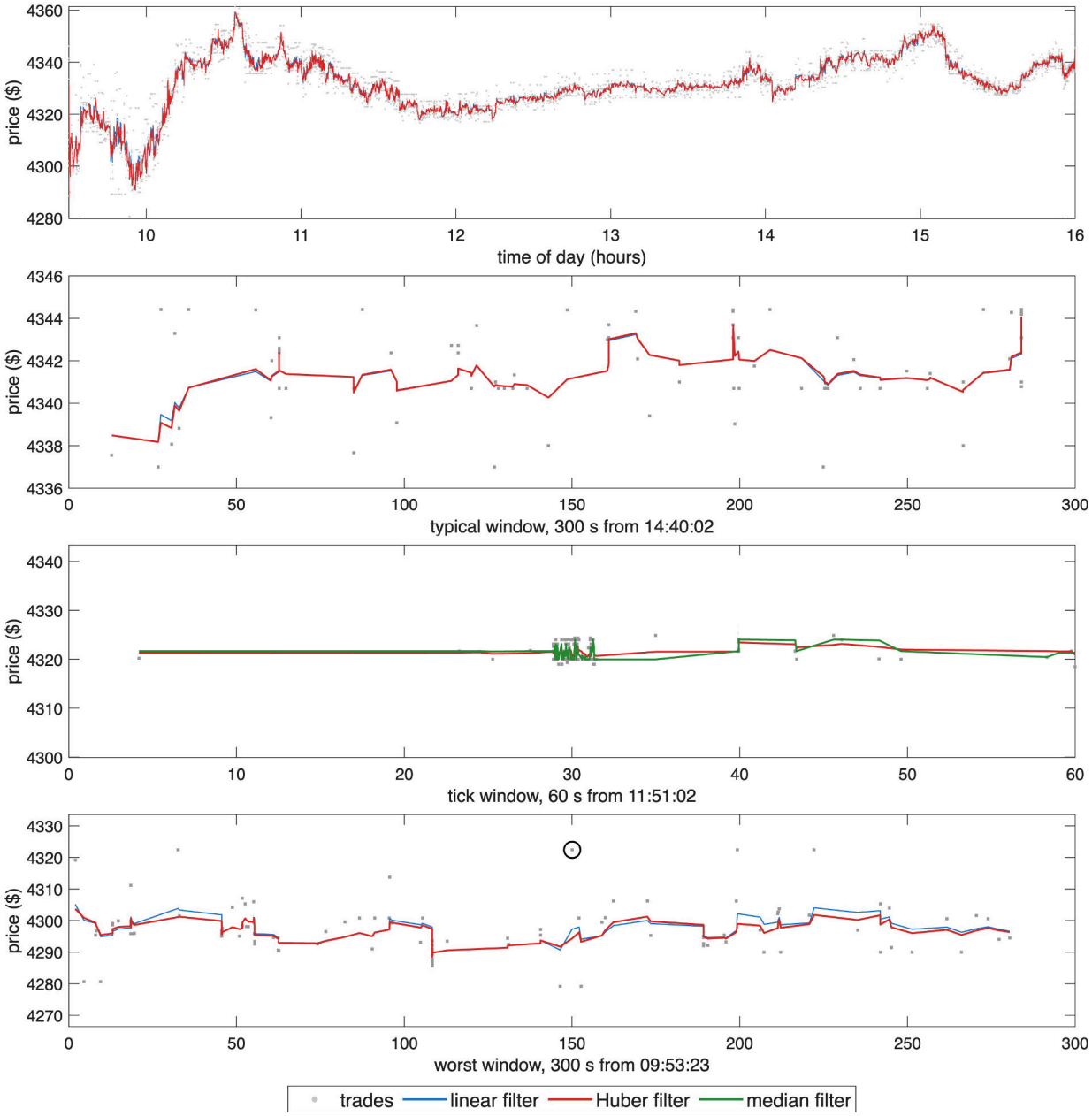

Figure 1: `BKNG` trades (grey points), the linear filter and the Huber filter, 16 October 2024. Top: full trading day. 2nd: 300-second window at the day's median trade count, from 14:40:02. 3rd: 60-second window at 11:51:02 in which the price reverses direction most often, with the median filter m_t added. Bottom: 300-second window around the largest Huber residual after 09:45, circled.

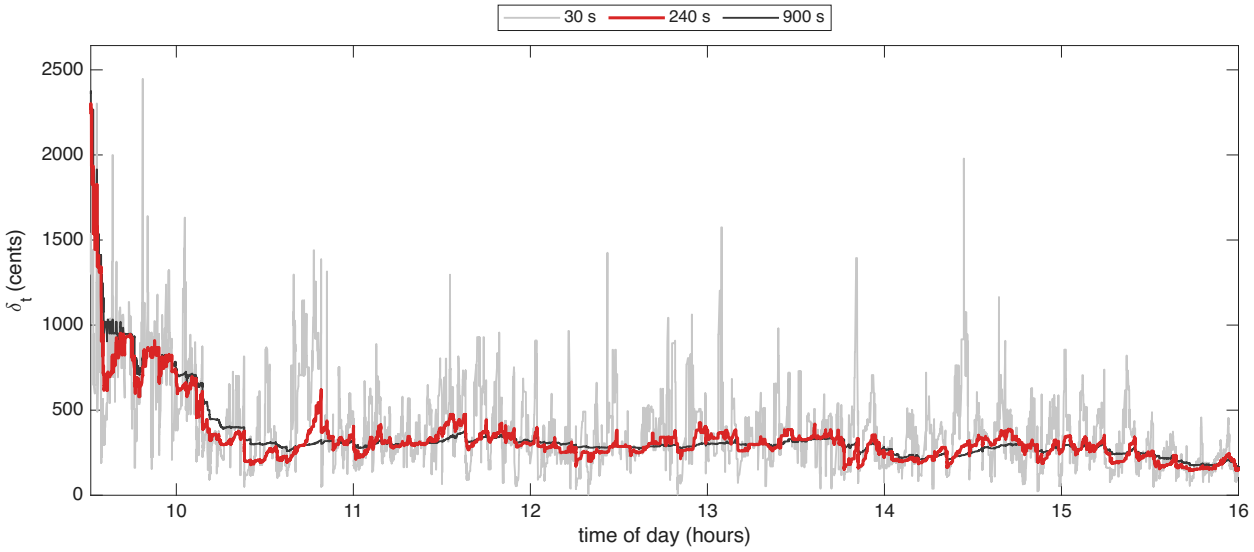

Figure 2: The Huber threshold δ_t in cents for `BKNG` on 16 October 2024, with half-lives of 30, 240, 900 seconds. The 240-second half-life, in red, is used.

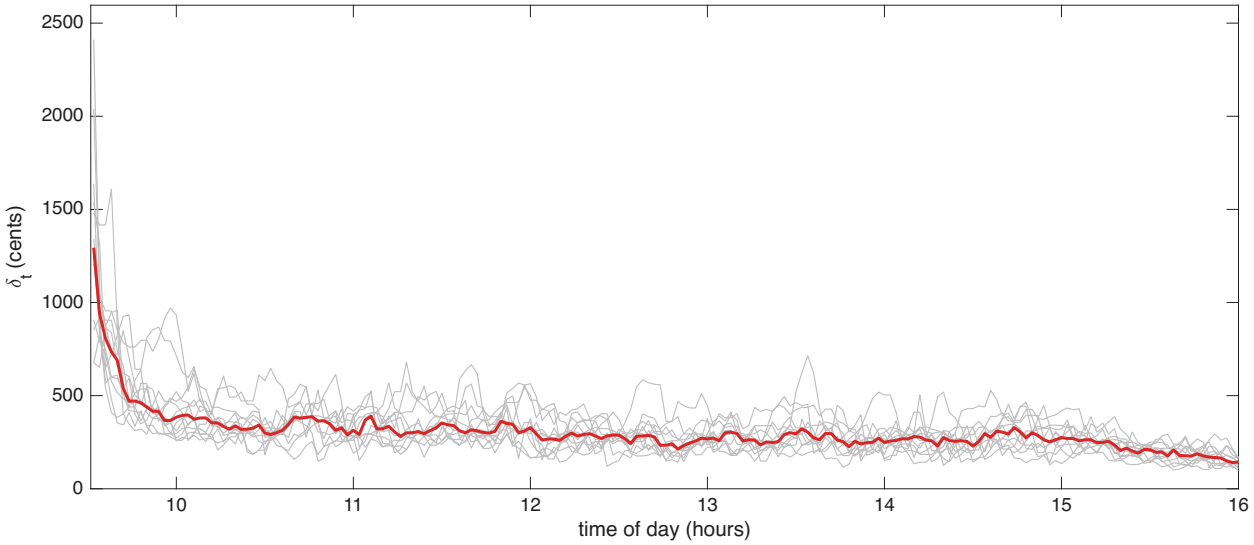

Figure 3: Huber threshold δ_t in cents through the trading day, for 12 `BKNG` trading days in Oct 2024 (grey lines) and their median (red line).

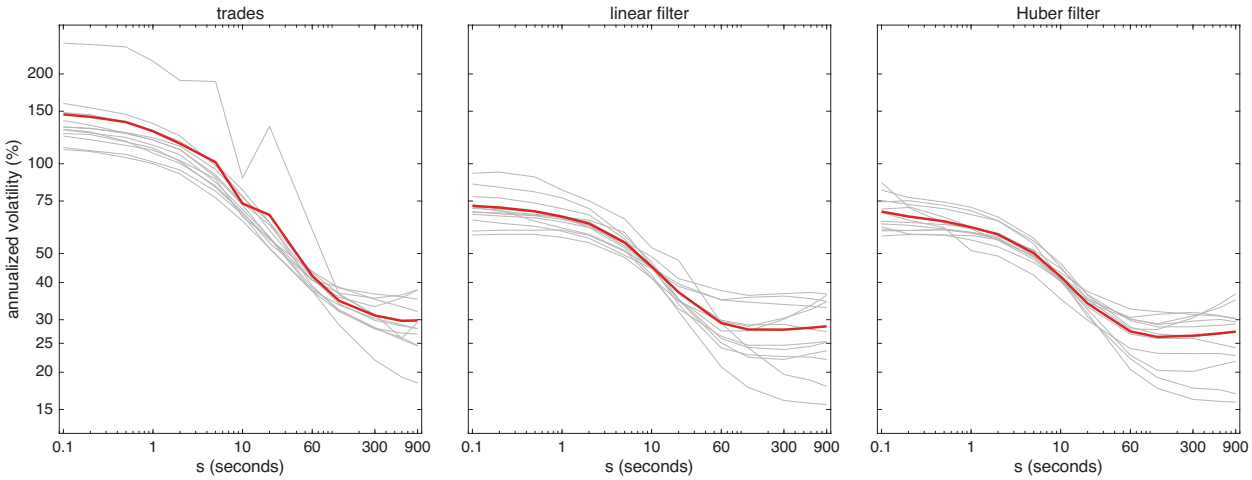

Figure 4: Volatility signature curves for `BKNG` over the 12 trading days in Oct 2024. The annualized vol $\hat{\sigma}100\sqrt{252}$ from the returns $\hat{X}_t - \hat{X}_{t,s}$ of Table 2, against the lag s in seconds, both axes on log scales, for: raw trades, linear filter & Huber filter. Grey lines are the individual trading days & the red line is the square root of the 12 day average of their squares. The linear & Huber returns carry the finite-sample correction $c_{t,s}$ of the paper; the raw trades are unaffected by it.

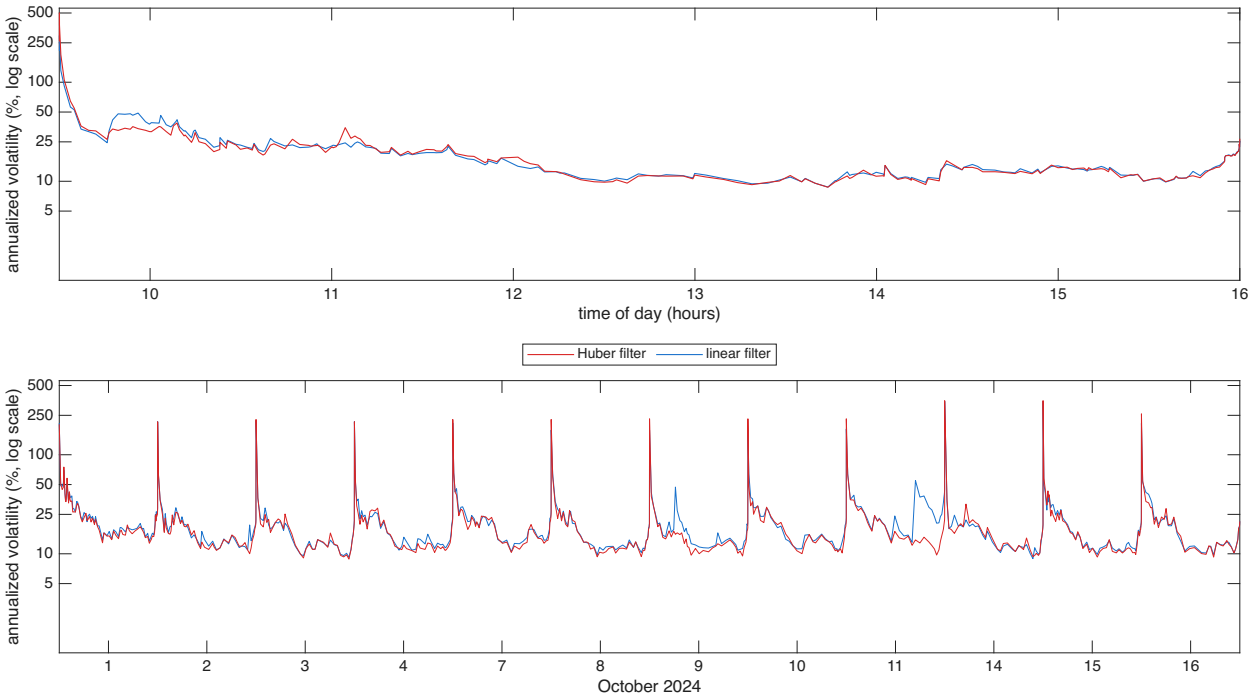

Figure 5: Filtered spot volatility for `BKNG` over 12 trading days in Oct 2024, the diurnally adjusted estimator $\hat{\sigma}(u)100\sqrt{252}$ at $s = 1$ second with hyperbolic weights, Mandelbrot parameter $a = 14$, with the input prices computed using the Huber & linear filters, shown as annualized volatility in percent on a log10 scale. Top: 16 October 2024, bottom: 12 days.

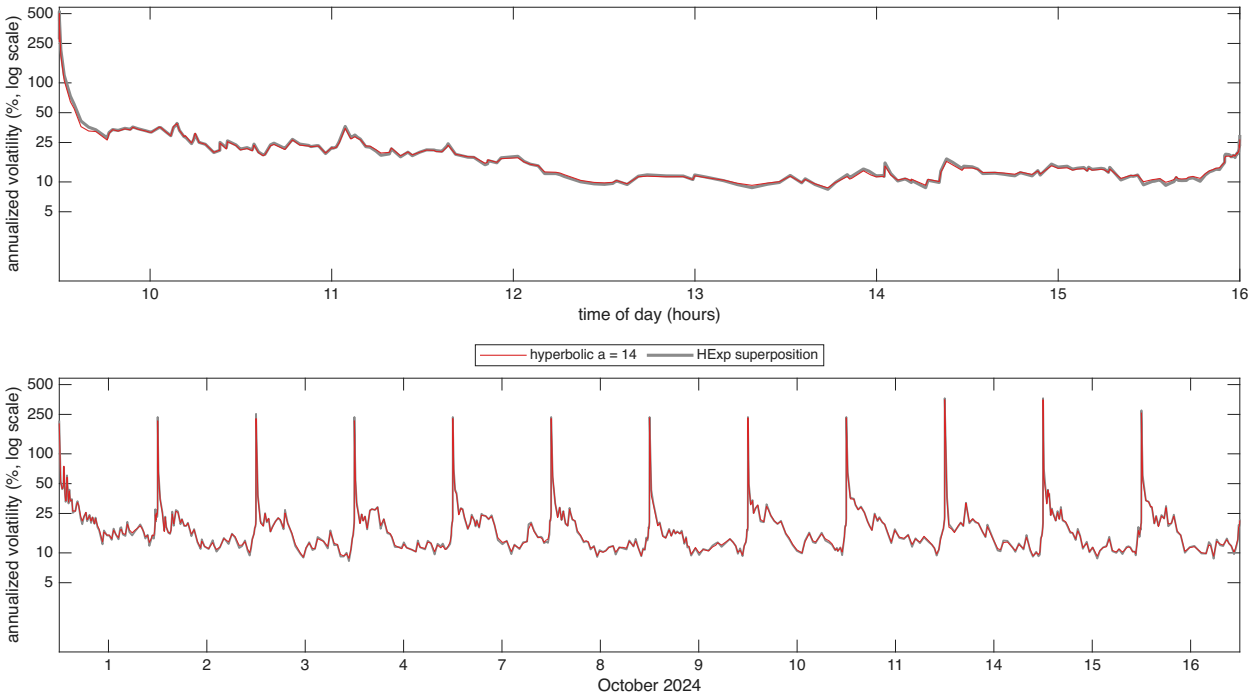

Figure 6: Hyperbolic against superposition weights for `BKNG` over 12 trading days in Oct 2024, the diurnally adjusted $\hat{\sigma}(u)100\sqrt{252}$ at $s = 1$ second from the Huber filter, shown as annualized vol in % on a log10 scale. The only difference between the 2 paths is the weight function: the hyperbolic weights with $a = 14$ in red, drawn on top, and the `HExp` superposition in grey, 4 exponential components with half lives of 10, 100, 1000, 10000 seconds. Hyperparameters minimize the QLIKE objective. Top: 16 October 2024, bottom: 12 days.

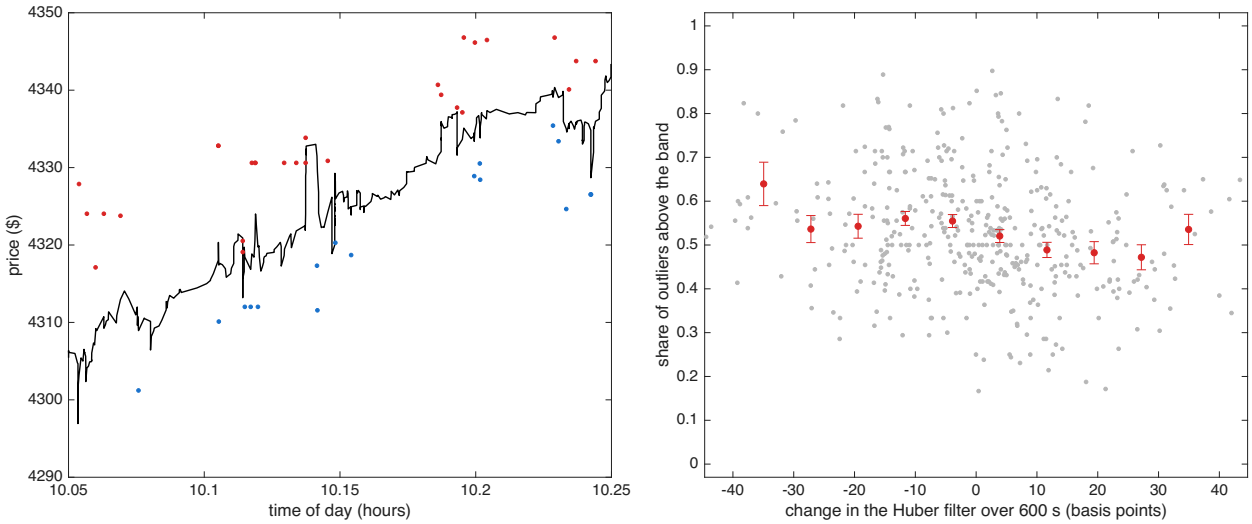

Figure 7: Outliers & robust price for `BKNG` on 16 October 2024. Left: the 12 minutes with the largest move in the Huber filter that day; outliers above $Y_t - \theta_t > \delta_{t-1}$ in red, outliers below $Y_t - \theta_t < -\delta_{t-1}$ in blue, Huber filter in black. Right: over the 12 trading days, the 600 second empirical conditional probability that outliers are above (rather than below) the Huber band against the 600 second change in the Huber filter in basis points: light points are single bins, the line & bars are means & s.e.

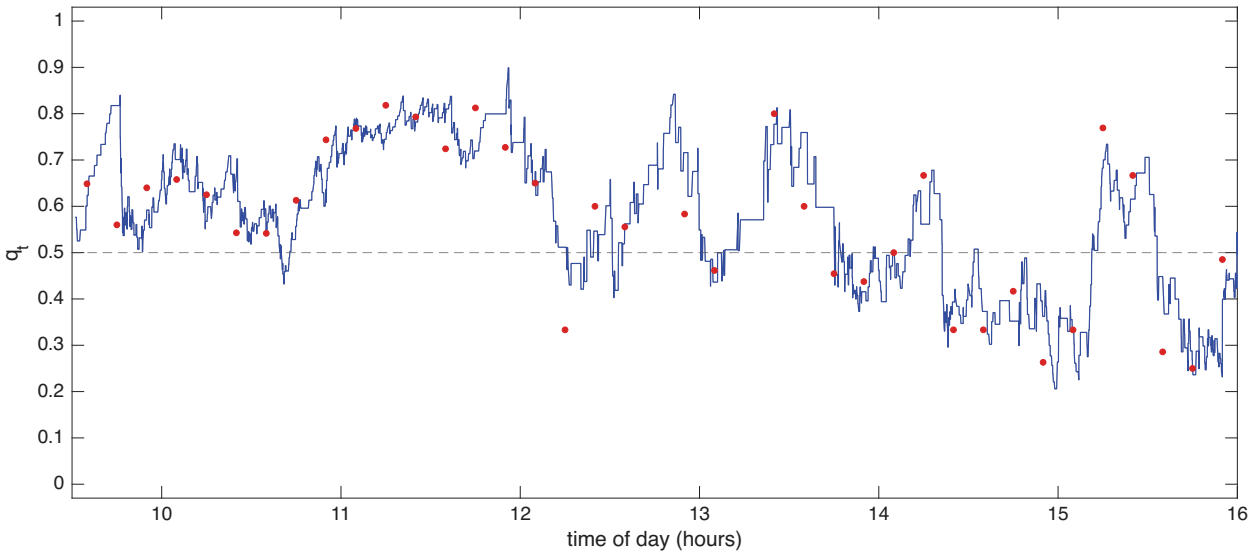

Figure 8: The directional conditional probability for BKNG, 16 October 2024. The filtered conditional probability q_t that an outlier lies above the robust Huber band, under exponential weights decaying in clock time with half life of 285 seconds; where fewer than two outliers remain in the weights it is drawn in grey. The dashed line marks 0.5. The red points are the empirical conditional probability using the 600 second bins of the previous figure.

At the horizons k = 3, 5, 10 and 30 the half-life minimizing the k-trade-ahead Huber forecast loss is 4, 4, 4 and 4 trades, within one grid point of the end of the 1 to 5 trade grid searched.
The direction half-life grid was extended beyond its usual end to 300 seconds, because the log-likelihood was still rising there.
On 16 October 2024, 32 per cent of the day's largest one per cent of residuals fall inside its first minute, against at most 30 per cent on any other day. Every number that day contributes to carries that burst.
The tail index of the residual cannot be estimated for every 10th trade, on none of the 12 days: what the row's mask or thinning leaves is too few residuals for the largest 0.1 to 1 per cent to hold enough order statistics.
The largest Huber residual on 16 October 2024 occurs at 09:30, within the opening minutes; the volatility comparison window is placed at the largest residual after 09:45, at 09:56.

Diagnostics.